SOLUTIONS
111

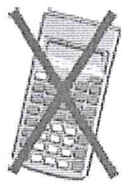
Edexcel GCSE

Mathematics (Linear) – 1MA0

PROOF

Materials required for examination
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser.
Tracing paper may be used.

Items included with question papers
Nil



Instructions

Use black ink or ball-point pen.
Fill in the boxes at the top of this page with your name, centre number and candidate number.
Answer all questions.
Answer the questions in the spaces provided – there may be more space than you need.
Calculators may be used.

Information

The marks for each question are shown in brackets – use this as a guide as to how much time to spend on **each** question.
Questions labelled with an **asterisk** (*) are ones where the quality of your written communication will be assessed – you should take particular care on these questions with your spelling, punctuation and grammar, as well as the clarity of expression.

Advice

Read each question carefully before you start to answer it.
Keep an eye on the time.
Try to answer every question.
Check your answers if you have time at the end.

1. The nth even number is $2n$.

The next even number after $2n$ is $2n + 2$

(a) Explain why.

Every alternate integer is even. As $2n$ is even $2n+1$ will be odd and so $2n+2$ is even.

(1)

(b) Write down an expression, in terms of n, for the next even number after $2n + 2$

$2n + 2 + 2 = 2n + 4$

$2n + 4$

(1)

(c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6

$2n + 2n+2 + 2n+4$

$= 6n + 6$

$= 6(n+1)$

↑ a multiple of 6.

(3)
(5 marks)

2. Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4, for all positive integer values of n.

$$(3n+1)^2 - (3n-1)^2$$

$$(3n+1)^2 = (3n+1)(3n+1)$$
$$= 9n^2 + 6n + 1$$

$$(3n-1)^2 = (3n-1)(3n-1)$$
$$= 9n^2 - 6n + 1$$

$$(3n+1)^2 - (3n+1)^2 = (9n^2 + 6n + 1) - (9n^2 - 6n + 1)$$
$$= 9n^2 + 6n + 1 - 9n^2 + 6n - 1$$
$$= 12n$$
$$= 4(3n)$$

$\uparrow$

which is a multiple of 4

(3 marks)

3. Prove, using algebra, that the sum of two consecutive whole numbers is always an odd number.

Two consecutive whole numbers

are n and $n+1$

$n + n+1 = 2n + 1$

$2n$ is a multiple of 2 so is even

$2n+1$ must be odd as it is one

more than an even number.

(3 marks)

4. Prove that

$$(2n+3)^2 - (2n-3)^2 \text{ is a multiple of } 8$$

for all positive integer values of n.

$$(2n+3)^2 = (2n+3)(2n+3)$$
$$= 4n^2 + 12n + 9$$

$$(2n-3)^2 = (2n-3)(2n-3)$$
$$= 4n^2 - 12n + 9$$

$$(2n+3)^2 - (2n-3)^2 = (4n^2 + 12n + 9) - (4n^2 - 12n + 9)$$
$$= 4n^2 + 12n + 9 - 4n^2 + 12n - 9$$
$$= 24n$$
$$= 8(3n)$$

↑

which is a multiple of 8

(3 marks)

*5. Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

Consecutive integers are n and $n+1$

Difference between the Squares of consecutive integers

$= (n+1)^2 - n^2$

$= (n+1)(n+1) - n^2$

$= n^2 + 2n + 1 - n^2$

$= 2n + 1$

Sum of 2 consecutive integes

$= n + n + 1$

$= 2n + 1$

So they are equal.

(4 marks)

6. Prove that $(5n + 1)^2 - (5n - 1)^2$ is a multiple of 5, for all positive integer values of n.

$$(5n+1)^2 = 25n^2 + 10n + 1$$

$$(5n-1)^2 = 25n^2 - 10n + 1$$

$$\begin{aligned}(5n+1)^2 - (5n-1)^2 &= (25n^2 + 10n + 1) - (25n^2 - 10n + 1)\\ &= 20n\\ &= 5(4n)\end{aligned}$$

↑

which is a multiple of 5

(3 marks)

7. If $2n$ is always even for all positive integer values of n, prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

Two consecutive even numbers are $2n$ and $2n+2$

Sum of their squares $= (2n)^2 + (2n+2)^2$

$= 4n^2 + 4n^2 + 8n + 4$

$= 8n^2 + 8n + 4$

$= 4(2n^2 + 2n + 1)$

$\uparrow$

which is a multiple of 4

(3 marks)

8. Prove that

$(n + 1)^2 - (n - 1)^2 + 1$ is always odd for all positive integer values of n.

$(n+1)^2 = n^2 + 2n + 1$

$(n-1)^2 = n^2 - 2n + 1$

$$(n+1)^2 - (n-1)^2 + 1 = (n^2 + 2n + 1) - (n^2 - 2n + 1) + 1$$
$$= n^2 + 2n + 1 - n^2 + 2n - 1 + 1$$
$$= 4n + 1$$

4n is a multiple of 4 so it must be even which means 4n+1 is odd.

(3 marks)

9. Prove algebraically that the sum of the squares of any two consecutive numbers always leaves a remainder of 1 when divided by 4.

consecutive numbers are n and $n+1$

$$n^2 + (n+1)^2$$

$$= n^2 + n^2 + 2n + 1$$

$$= 2n^2 + 2n + 1$$

$$= 2n(n+1) + 1$$

$n(n+1)$ is the product of 2 consecutive numbers. As one of them is even the product must be even.

$2n(n+1)$ is $2 \times$ an even number which has to be a multiple of 4

So $2n(n+1) + 1$ is a multiple of 4 plus 1 and will leave a remainder of 1 when divided by 4

(4 marks)